# mRCWA Formulation

Formulation organized by Po-Han Chiu (Hans Chiu)

## Reference: EMPossible

* Formulation of Rigorous Coupled‐Wave Analysis (RCWA)  
  <https://empossible.net/wp-content/uploads/2019/08/Lecture-7a-RCWA-Formulation.pdf>
* R. C. Rumpf, “IMPROVED FORMULATION OF SCATTERING MATRICES FOR SEMI-ANALYTICAL METHODS THAT IS CONSISTENT WITH CONVENTION,” PIER B, vol. 35, pp. 241–261, 2011, doi: 10.2528/PIERB11083107.
* Lecture 19 (CEM) -- Formulation of Rigorous Coupled-Wave Analysis  
  <https://www.youtube.com/watch?v=LEWTvwrYxiI&t=1s&ab_channel=EMPossible>

# Maxwell’s equation

Let’s start with the most fundamental four equations:

|  |  |
| --- | --- |
| Gauss’s law |  |
| Gauss’s law for magnetism |  |
| Maxwell-Faraday |  |
| Ampere with Maxwell’s addition |  |

Assume there is no source: and under linear material

We have:

Assume , 🡪

We want to normalize the equation so that:

Therefore, 🡪

After normalization of 🡪

So we got the normalized Maxwell equation:

Expanding the curl operation:

We have six equation.

# Field Expansion

is the amplitude coefficient, complex number

Using as an example,

Let ,

Combine the above to get:

For each m, n:

Write in matrix form:

Where and are vector, is the diagonal matrix:

is the number of modes for each term will correspond to a certain mode

Which means, is the flattened , is the flattened , is the flattened, and so on.

And becomes a convolution matrix:

is the number of modes, where index ,

# Compare to the DFT algorithm:

Our expansion is:

DFT definition is:

Where is the number of sample points, is the coordinate in the index scale:

Therefore:

By comparison, the term we need , is the DFT term divide by the number of sample points :

Also, By DFT Definition:

can be any integer; let

Therefore,

Mod by it’s safe to access any coefficient even if is out of bounds.

Original six equations:

In matrix form:

Normalize by :

Solve for and :

Substitute back to the equation:

We get:

# Matrix Equation

In matrix form:

Similarly:

We have:

And differential equation:

Solving:

General Solution:

Where is the eigenvalue matrix of , is the eigenvector matrix of , is the coefficient.

Further, separate the solution into forward and backward waves:

And assume a similar solution for magnetic field:

Recall :

# Boundary Condition

We can formulate the solution:

And start to match the boundary condition:

Assume a structure with three layers, and their eigenmodes are:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Layer 1 | Layer N | Layer 2 |
| Eigen modes |  |  |  |
| Field coefficient |  |  |  |
| Thickness | 0 |  | 0 |

Between Layer 1 and Layer N,

Between Layer N and Layer 2,

Therefore:

The inverse of the matrix:

Verify:

Therefore:

# Scattering matrix

Simplify:

Let

Write into the form of a scattering matrix:

Let 🡪

|  |  |
| --- | --- |
| 1. From the matrix | 1. Solve for S21 and S11 |
| 1. Substitute back into 1 | 1. Rearrange, bringing out S21 and S11 |

We have:

Similarly:

Summary:

When , :

Reflection side, when 🡪

Transmision side, when 🡪

# Scattering matrix operation

Assume a structure with three layers connected by two scattering matrices, A and B

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |
|  |  |  |  |  |
| 🡨 | 🡨 | 🡨 | 🡨 | 🡨 |

Combine into one scattering matrix:

Case 1:

|  |  |
| --- | --- |
| 1. From matrix A | 1. From matrix B |
| 1. Substitute A2 and B1: | 1. Solve for C2: |

Case 2:

|  |  |
| --- | --- |
| 1. From matrix A | 1. From matrix B |
| 1. Substitute A2 and B1: | 1. Solve for C2: |

Summary:

Define as , called the Redheffer Star Product.

# Final solution

# Finding z component:

Power:

Power Ratio

# Special Case: No Magnetic

# Special Case: Homogeneous Layer

Since it is a diagonal matrix, it’s already diagonalized.

Interesting fact:

Recall solution for the electric field is:

Note that the relation between E and H is ,

Also, for magnetic, the other solution matches the result given above:

In free space:

# Boundary Condition – Improved Formulation

We can formulate the solution:

And start to match the boundary condition:

Assume a structure with three layers, and their eigenmodes are:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Layer 1 | Layer N | Layer 2 |
| Eigen modes |  |  |  |
| Field coefficient |  |  |  |
| Thickness | 0 |  | 0 |

Between Layer 1 and Layer N,

Between Layer N and Layer 2,

Therefore:

The inverse of the matrix:

Therefore:

# Scattering matrix

Let

Write into the form of a scattering matrix:

Let 🡪

|  |  |
| --- | --- |
| 1. From the matrix | 1. Solve for S21 and S21 |
| 1. Substitute back into 1 | 1. Rearrange, bringing out S11 and S21 |

We have:

🡪 Compare to original, this result swaps A and B